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Unique equilibrium in rent-seeking contests with a continuum of types[☆]

Christian Ewerhart

Department of Economics, Schönberggasse 1, 8001 Zurich, Switzerland

HIGHLIGHTS

- Considered are rent-seeking contests with continuous and independent types.
- Contestants may be ex-ante asymmetric.
- It is shown that there exists a unique pure-strategy Nash equilibrium.

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ABSTRACT

It is shown that rent-seeking contests with continuous and independent type distributions possess a unique pure-strategy Nash equilibrium.

1. Introduction

While rent-seeking contests with continuous and independent type distributions are quite interesting, basic issues such as existence and uniqueness of a pure-strategy Nash equilibrium (PSNE) have been addressed only partially.[1] Indeed, previous work on the issue of existence focused either on symmetric contests (Fey, 2008; Ryvkin, 2010) or on the case of a continuous technology (Wasser, 2013a,b). Moreover, little general was known about the uniqueness of the equilibrium.

Below, it is shown that in any rent-seeking contest with independent and continuous types, there exists a unique PSNE.[2] The contest success function merely needs to be of the logit form with concave impact functions, and players' private information may relate to either marginal costs or valuations. The result holds even when the contest is ex-ante asymmetric,[3] so that the equilibrium may entail inactive types.[4] Moreover, no restriction is imposed on the shape of the type distributions. Generally, existence ensures consistency of a model, whereas uniqueness strengthens numerical analyses, theoretical results, and experimental findings.

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E-mail addresses: christian.ewerhart@iew.uzh.ch, christian.ewerhart@econ.uzh.ch.

1 Generally, in games of incomplete information, the PSNE refers to strategic optimization at the ex-ante stage (Athey, 2001). See Section 2 for a formal definition and the Appendix for further discussion.

2 Uniqueness means here that for any given player, any two PSNE strategies differ at most on a null set. This corresponds to the strongest form of uniqueness for PSNE.

3 Ex-ante asymmetry may be reflected, e.g., in heterogeneous distributions of marginal costs, heterogeneous distributions of valuations, or in heterogeneous economies of scale.

4 Wärneryd (2003) explicitly allows for inactive types in a common-value setting.

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The rest of the paper is structured as follows. Section 2 describes the set-up. Existence is dealt with in Section 3. Section 4 discusses uniqueness. A numerical illustration can be found in Section 5. Section 6 concludes. An Appendix contains technical lemmas.

2. Set-up

There are $N \geq 2$ players. Each player $i = 1, \ldots, N$ observes a signal (or type) c_i, drawn from an interval $D_i = [\underline{c}_i, \overline{c}_i]$, where $0 < \underline{c}_i < \overline{c}_i$. Signals are independent across players. Moreover, player i does not observe the signal c_j of any other player $j \neq i$. The distribution function of player i's signal is denoted by $F_i = F_i(c_i)$. Each player i chooses a level of activity $y_i \geq 0$ at cost $g_i(y_i)$. It is assumed that $g_i(0) = 0$, and that g_i is twice continuously differentiable on $\mathbb{R}_+$, with $g_i' > 0$ on $\mathbb{R}_{++}$, and $g_i'' \geq 0$. Player i's payoff is $\Pi_i(y_i, y_{-i}, c_i) = p_i(y_i, y_{-i}) - c_i g_i(y_i)$, where $p_i(y_i, y_{-i}) = y_i/(y_i + \sum_{j\neq i} y_j)$ if $y_i + \sum_{j\neq i} y_j > 0$, and $p_i(y_i, y_{-i}) = 1/N$ otherwise.[5]

A strategy for player i is a (measurable) mapping $\sigma_i : D_i \to \mathbb{R}_+$. Denote by S_i the set of strategies for player i. For a profile $\sigma_{-i} = \{\sigma_j\}_{j\neq i} \in S_{-i} = \prod_{j\neq i} S_j$, and a type $c_i \in D_i$, player i's interim expected payoff is given by $\overline{\Pi}_i(y_i, \sigma_{-i}, c_i) = \int_{D_{-i}} \Pi_i (y_i, \sigma_{-i}(c_{-i}), c_i) dF_{-i}(c_{-i})$, where $D_{-i} = \prod_{j\neq i} D_j$, $\sigma_{-i}(c_{-i}) = \{\sigma_j(c_j)\}_{j\neq i}$, and $dF_{-i}(c_{-i}) = \prod_{j\neq i} dF_j(c_j)$. A *Bayesian Nash equilibrium* (*BNE*) is a profile $\sigma^* = \{\sigma_i^*\}_{i=1}^N \in S = \prod_{i=1}^N S_i$ such that $\overline{\Pi}_i(\sigma_i^*(c_i), \sigma_{-i}^*, c_i) \geq \overline{\Pi}_i(y_i, \sigma_{-i}^*, c_i)$ for any $i = 1, \ldots, N$, any $c_i \in D_i$, and any $y_i \geq 0$. A *pure-strategy Nash equilibrium* (*PSNE*) is a profile $\sigma^* \in S$ such that for any $i = 1, \ldots, N$, and for almost any $c_i \in D_i$, the inequality $\overline{\Pi}_i(\sigma_i^*(c_i), \sigma_{-i}^*, c_i) \geq \overline{\Pi}_i(y_i, \sigma_{-i}^*, c_i)$ holds for any $y_i \geq 0$.[6]

3. Existence

This section builds on prior work by Fey (2008), Ryvkin (2010) and Wasser (2013a). Existence is shown first for the *ε-constrained contest*, for $\varepsilon > 0$, in which each player $i = 1, \ldots, N$ may use only strategies with values in $[\varepsilon, \infty)$.

Lemma 3.1. *There is a level of activity $E > 0$ such that, for any sufficiently small $\varepsilon > 0$, there exists a BNE σ^ε in the ε-constrained contest such that each player i's strategy σ_i^ε is continuous, monotone, and bounded by E.*

Proof. Since costs are strictly increasing and convex, there is an $E > 0$ such that any $y_i > E$ is suboptimal. Moreover, $\overline{\Pi}_i$ exhibits decreasing differences in y_i and c_i. Hence, existence of a monotone PSNE $\widetilde{\sigma}^\varepsilon$ in the ε-constrained contest follows from Athey (2001, Cor. 2.1). Note now that type c_i's ε-constrained problem, $\max_{y_i\geq\varepsilon} \overline{\Pi}_i(y_i, \widetilde{\sigma}_{-i}^\varepsilon, c_i)$, has a unique solution $y_i = \sigma_i^\varepsilon(c_i)$. Indeed, if $\widetilde{\sigma}_{-i}^\varepsilon(c_{-i}) \neq 0$ with positive probability, then $\overline{\Pi}_i(\cdot, \widetilde{\sigma}_{-i}^\varepsilon, c_i)$ is strictly concave on $[\varepsilon, E]$, while otherwise, the unique solution is $y_i = \varepsilon$. Hence, $\sigma_i^\varepsilon(c_i) = \widetilde{\sigma}_i^\varepsilon(c_i)$ with probability one, for any $i = 1, \ldots, N$. This implies that $\sigma_i^\varepsilon(c_i)$ is also type c_i's best response to σ_{-i}^ε, for any $i = 1, \ldots, N$, and any $c_i \in D_i$. Thus, $\sigma^\varepsilon = (\sigma_1^\varepsilon, \ldots, \sigma_N^\varepsilon)$ is a BNE in the ε-constrained contest. Clearly, each σ_i^ε is monotone. Finally, continuity of σ_i^ε follows from Berge's Theorem, as $\overline{\Pi}_i(\cdot, \sigma_{-i}^\varepsilon, \cdot)$ is continuous on the compact set $[\varepsilon, E] \times D_i$. □

Consider now a sequence $\{\varepsilon_m\}_{m=1}^\infty$ such that $\varepsilon_m \searrow 0$, and select a BNE σ^m in the ε_m-constrained contest for each $m \in \mathbb{N}$, with the properties specified in the previous lemma.

Lemma 3.2. *The sequence $\{\sigma^m\}_{m=1}^\infty$ has a uniformly converging subsequence.*

Proof. In view of Lemma 3.1 and the Theorem of Arzelà–Ascoli, it suffices to find a $\lambda > 0$ such that σ_i^m has everywhere a slope exceeding $-\lambda$ for any $m \in \mathbb{N}$ and any i. In terms of the transformed choice variable $y_i^\lambda = y_i + \lambda c_i$, a type c_i's expected payoff in σ^m may be written as

$$\overline{\Pi}_i^\lambda(y_i^\lambda, \sigma_{-i}^m, c_i) = \int_{D_{-i}} \frac{(y_i^\lambda - \lambda c_i) dF_{-i}(c_{-i})}{y_i^\lambda - \lambda c_i + \sum_{j\neq i} \sigma_j^m(c_j)} - c_i g_i(y_i^\lambda - \lambda c_i), \tag{1}$$

provided that $y_i^\lambda - \lambda c_i = y_i > 0$. Hence, for λ sufficiently large, the cross-partial

$$\frac{\partial^2 \overline{\Pi}_i^\lambda}{\partial y_i^\lambda \partial c_i} = \int_{D_{-i}} \frac{2\lambda \sum_{j\neq i} \sigma_j^m(c_j) dF_{-i}(c_{-i})}{\left(y_i + \sum_{j\neq i} \sigma_j^m(c_j)\right)^3} - g_i'(y_i) + \underbrace{c_i \lambda g_i''(y_i)}_{\geq 0} \tag{2}$$

$$\geq \frac{2\lambda}{NE} \int_{D_{-i}} \frac{\sum_{j\neq i} \sigma_j^m(c_j) dF_{-i}(c_{-i})}{\left(y_i + \sum_{j\neq i} \sigma_j^m(c_j)\right)^2} - g_i'(y_i) \tag{3}$$

$$\geq \left(\frac{2\lambda \underline{c}_i}{NE} - 1\right) g_i'(y_i) \tag{4}$$

is seen to be positive in the range of c_i where $y_i = \sigma_i^m(c_i) > 0$. Thus, for λ large, y_i^λ is weakly increasing in c_i, which proves the claim. □

By Lemma 3.2, one may assume w.l.o.g. that $\{\sigma^m\}_{m=1}^\infty$ converges uniformly to some $\sigma^* \in S$. Next, it is shown that in σ^*, at least one player is active with probability one.

Lemma 3.3. *There is some player i such that $\sigma_i^*(c_i) > 0$ with probability one.*

Proof. Suppose that for each i, there is a set $\mathcal{D}_i \subseteq D_i$ of positive measure such that $\sigma_i^*(c_i) = 0$ for all $c_i \in \mathcal{D}_i$. Then, by uniform convergence, there exists, for any $\varepsilon > 0$, an $m_0 = m_0(\varepsilon)$ such that $\sigma_i^m(c_i) < \varepsilon$ for any i, any $c_i \in \mathcal{D}_i$, and any $m \geq m_0$. But, from the Kuhn–Tucker condition for type c_i in the ε_m-constrained contest,

$$0 \geq \int_{\mathcal{D}_{-i}} \frac{\sum_{j\neq i} \sigma_j^m(c_j) dF_{-i}(c_{-i})}{\left(\sigma_i^m(c_i) + \sum_{j\neq i} \sigma_j^m(c_j)\right)^2} - c_i g_i'(E), \tag{5}$$

where $\mathcal{D}_{-i} = \prod_{j\neq i} \mathcal{D}_j$. Integrating over $\mathcal{D}_i$, and subsequently summing over $i = 1, \ldots, N$, one obtains

$$0 \geq \int_{\mathcal{D}} \frac{(N-1) dF(c)}{\sum_{i=1}^N \sigma_i^m(c_i)} - \sum_{i=1}^N g_i'(E) \int_{\mathcal{D}_i} c_i dF_i(c_i), \tag{6}$$

where $\mathcal{D} = \prod_{i=1}^N \mathcal{D}_i$ and $dF(c) = \prod_{i=1}^N dF_i(c_i)$. For ε small, however, this is impossible. □

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5 As usual, a simple change of variables allows to capture other types of contest success functions and other forms of uncertainty, e.g., about valuations. Cf. Ryvkin (2010).

6 As shown in the Appendix, this amounts to the standard definition.

The following is the first main result of this paper.

Theorem 3.4. *In the unconstrained contest, σ^* is a PSNE in continuous and monotone strategies.*

Proof. Fix a player $i \in \{1, \ldots, N\}$. For any $m \in \mathbb{N}$, since σ^m is a BNE in the ε_m-constrained contest, $\overline{\Pi}_i(\sigma_i^m(c_i), \sigma_{-i}^m, c_i) \geq \overline{\Pi}_i(y_i, \sigma_{-i}^m, c_i)$ for any $c_i \in D_i$ and any $y_i \geq \varepsilon_m$. Therefore, if the event $\sigma_{-i}^*(c_{-i}) = 0$ is null, letting $m \to \infty$ implies $\overline{\Pi}_i(\sigma_i^*(c_i), \sigma_{-i}^*, c_i) \geq \overline{\Pi}_i(y_i, \sigma_{-i}^*, c_i)$ via Lebesgue's theorem, for any $c_i \in D_i$ and any $y_i > 0$. Suppose next that $\sigma_{-i}^*(c_{-i}) = 0$ with positive probability. Then, by Lemma 3.3, $\sigma_i^*(c_i) > 0$ with probability one. Let $c_i \in D_i$ with $\sigma_i^*(c_i) > 0$. If $y_i > 0$, then the argument proceeds as above. To complete the proof, note that $\overline{\Pi}_i(\cdot, \sigma_{-i}^*, c_i)$ is l.s.c., so that $y_i = 0$ cannot be the only profitable deviation for c_i. □

4. Uniqueness

Consider two PSNE σ^* and σ^{**} in the unconstrained contest such that, for some player i, the event $\sigma_i^*(c_i) \neq \sigma_i^{**}(c_i)$ has positive probability. Then, as noted below, σ^* and σ^{**} must differ in an essential way for at least two players.

Lemma 4.1. *There are players $i \neq j$ such that each of the independent events $\sigma_i^*(c_i) \neq \sigma_i^{**}(c_i)$ and $\sigma_j^*(c_j) \neq \sigma_j^{**}(c_j)$ has positive probability.*

Proof. Suppose there is some i such that $\sigma_{-i}^*(c_{-i}) = \sigma_{-i}^{**}(c_{-i})$ with probability one. Then, $\overline{\Pi}_i(\cdot, \sigma_{-i}^*, c_i) = \overline{\Pi}_i(\cdot, \sigma_{-i}^{**}, c_i)$ for any $c_i \in D_i$. Thus, $\sigma_i^*(c_i) = \sigma_i^{**}(c_i)$ with probability one, which is a contradiction. □

The following is the second main result of this paper.

Theorem 4.2. *The PSNE in the unconstrained contest is unique.*

Proof. Following Rosen (1965), write $\sigma^{*,s} = (1-s)\sigma^* + s\sigma^{**}$ for $0 \leq s \leq 1$, and consider

$$\Phi_s = \sum_{i=1}^{N} \int_{D_i} \overline{\pi}_i(\sigma^{*,s}, c_i) \left(\sigma_i^{**}(c_i) - \sigma_i^*(c_i)\right) dF_i(c_i) \tag{7}$$

for $s = 0, 1$, where $\overline{\pi}_i(\sigma, c_i) = \partial \overline{\Pi}_i(\sigma_i(c_i), \sigma_{-i}, c_i)/\partial y_i$ denotes type c_i's marginal expected payoff at a profile $\sigma \in S$.[7] From the Kuhn–Tucker conditions, $\overline{\pi}_i(\sigma^*, c_i) \leq 0$ for almost any $c_i \in D_i$; moreover, $\sigma_i^*(c_i) = 0$ if $\overline{\pi}_i(\sigma^*, c_i) < 0$. It follows that $\Phi_0 \leq 0$, and similarly, $\Phi_1 \geq 0$. To provoke a contradiction, it will be shown now that $\Phi_1 - \Phi_0 < 0$. Denote by $\pi_i(\sigma, c_i, c_{-i}) = \partial \Pi_i(\sigma_i(c_i), \sigma_{-i}(c_{-i}), c_i)/\partial y_i$ type c_i's marginal ex-post payoff at $\sigma \in S$, when facing $c_{-i} \in D_{-i}$. Then, by Lemma A.2 in the Appendix,

$$\Phi_1 - \Phi_0 = \int_D \sum_{i=1}^{N} (\pi_i(\sigma^{**}, c_i, c_{-i}) - \pi_i(\sigma^*, c_i, c_{-i})) z_i(c_i) dF(c) \tag{8}$$

$$= \int_D \sum_{i=1}^{N} \left\{ \int_0^1 \frac{\partial \pi_i(\sigma^{*,s}, c_i, c_{-i})}{\partial s} z_i(c_i) ds \right\} dF(c), \tag{9}$$

where $z_i(c_i) = \sigma_i^{**}(c_i) - \sigma_i^*(c_i)$. An application of the chain rule delivers

$$\frac{\partial \pi_i(\sigma^{*,s}, c_i, c_{-i})}{\partial s} = \sum_{j=1}^{N} \frac{\partial^2 p_i(\sigma_i^{*,s}(c_i), \sigma_{-i}^{*,s}(c_{-i}))}{\partial y_i \partial y_j} z_j(c_j) - c_i \underbrace{g_i''(\sigma_i^{*,s}(c_i))}_{\geq 0} z_i(c_i), \tag{10}$$

for any i, any $c_i \in D_i$, and any $c_{-i} \in D_{-i}$. It follows that

$$\Phi_1 - \Phi_0 \leq \int_D \left(\int_0^1 \left(\sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial^2 p_i(\sigma_i^{*,s}(c_i), \sigma_{-i}^{*,s}(c_{-i}))}{\partial y_i \partial y_j} \times z_i(c_i) z_j(c_j) \right) ds \right) dF(c). \tag{11}$$

One can verify, however, that

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\partial^2 p_i(y_i, y_{-i})}{\partial y_i \partial y_j} z_i z_j \tag{12}$$

$$= -\sum_{i=1}^{N} \frac{2Y_{-i}}{Y^3} z_i^2 + \sum_{i=1}^{N} \sum_{j \neq i} \frac{Y - 2Y_{-i}}{Y^3} z_i z_j \tag{13}$$

$$= -\frac{2}{Y^3} \sum_{i=1}^{N} Y_{-i} z_i^2 - \frac{2}{Y^3} \sum_{i=1}^{N} \sum_{j>i} \sum_{k \neq i,j} y_k z_i z_j \tag{14}$$

$$= -\frac{1}{Y^3} \sum_{i=1}^{N} Y_{-i} z_i^2 - \frac{1}{Y^3} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k \neq i,j} y_k z_i z_j \tag{15}$$

$$= -\frac{1}{Y^3} \sum_{i=1}^{N} (z_i^2 Y_{-i} + y_i Z_{-i}^2) \leq 0 \tag{16}$$

for any $(y_1, \ldots, y_N) \in \mathbb{R}_+^N \setminus \{0\}$ and any $(z_1, \ldots, z_N) \in \mathbb{R}^N$, where $Y = \sum_{i=1}^N y_i$, $Y_{-i} = \sum_{j \neq i} y_j$, and $Z_{-i} = \sum_{j \neq i} z_j$. Moreover, $z_i^2 Y_{-i} = z_i(c_i)^2 \sum_{j \neq i} \sigma_j^{*,s}(c_j)$ is positive for any $s \in (0, 1)$ if $\sigma_i^*(c_i) \neq \sigma_i^{**}(c_i)$ and $\sigma_j^*(c_j) \neq \sigma_j^{**}(c_j)$ for some $j \neq i$. Thus, by Lemma 4.1, $\Phi_1 - \Phi_0 < 0$. □

5. Numerical illustration

Fig. 1 shows PSNE strategies in a two-player lottery contest, where types are distributed uniformly on $D_1 = [0.01, 1.01]$ and $D_2 = [0.51, 5.51]$, respectively. Note that player 2 remains inactive for $c_2 > c_2^* \approx 4.21$.

6. Concluding remarks

While this paper has focused on the existence and uniqueness of a PSNE in asymmetric rent-seeking contests, it follows from the proofs that also any of the BNE studied by Fey (2008) and Ryvkin (2010) is unique.

Appendix. Technical lemmas

Lemma A.1. *A profile $\sigma^* \in S$ is a PSNE in the unconstrained contest if and only if $\int_D \Pi_i(\sigma_i^*(c_i), \sigma_{-i}^*(c_{-i}), c_i) dF(c) \geq \int_D \Pi_i(\widehat{\sigma}_i(c_i), \sigma_{-i}^*(c_{-i}), c_i) dF(c)$ for any $i = 1, \ldots, N$, and any $\widehat{\sigma}_i \in S_i$.*

Proof. Let σ^* be a PSNE, and consider a deviation $\widehat{\sigma}_i \in S_i$ for some player i. Then, $\overline{\Pi}_i(\sigma_i^*(c_i), \sigma_{-i}^*, y_i) \geq \overline{\Pi}_i(\widehat{\sigma}_i(c_i), \sigma_{-i}^*, c_i)$ for almost any $c_i \in D_i$. Integrating over D_i, the assertion follows via Fubini's theorem. Conversely, suppose that σ^* is not a PSNE. Then, there is a player i and a set $\mathcal{D}_i \subseteq D_i$ of positive measure such that $\sigma_i^*(c_i)$ is not a best response to σ_{-i}^* for c_i, for any $c_i \in \mathcal{D}_i$. Define $\widehat{\sigma}_i(c_i)$ as c_i's best response to σ_{-i}^* if it exists; otherwise as $\sigma_i^*(c_i)/2$ if $\sigma_i^*(c_i) > 0$, and as $\mathrm{pr}\{\sigma_{-i}^*(c_{-i}) = 0\}/(2\overline{c}_i g_i'(E))$ if $\sigma_i^*(c_i) = 0$. Then $\widehat{\sigma}_i$ is a profitable deviation. □

Lemma A.2. *Let $\sigma^* \in S$ be a PSNE in the unconstrained contest. Then, for almost any $c_i \in D_i$, the function $\pi_i(\sigma^*, c_i, \cdot)$ is integrable,*

[7] It is shown in the Appendix that Φ_0 and Φ_1 are well-defined.

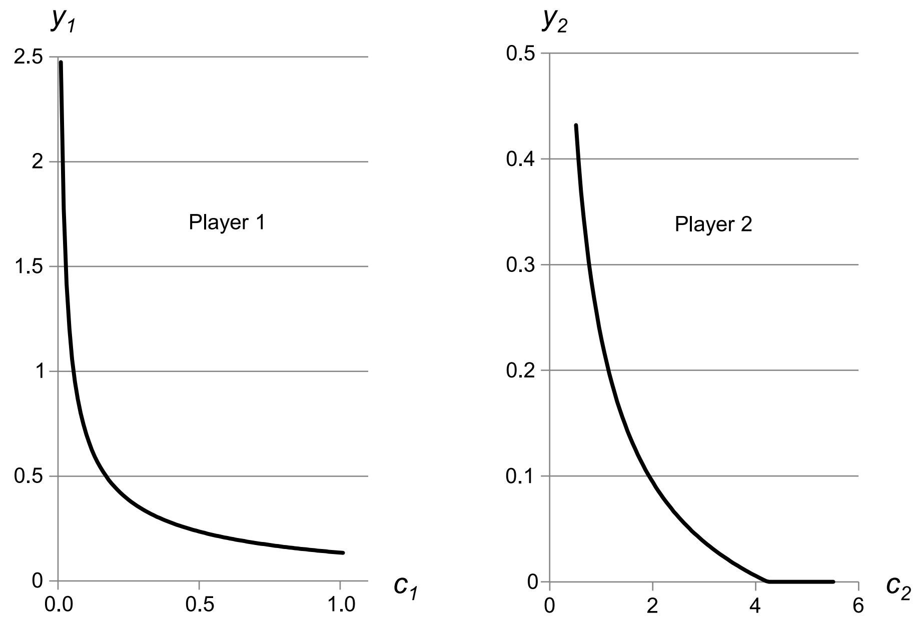

Fig. 1. An equilibrium involving inactive types.

with $\overline{\pi}_i(\sigma^*, c_i) = \int_{D_{-i}} \pi_i(\sigma^*, c_i, c_{-i})dF_{-i}(c_{-i})$. *Moreover,* $\overline{\pi}_i(\sigma^*, \cdot)$ *is integrable.*

Proof. The first claim is obvious if $\sigma_i^*(c_i) > 0$ for almost any $c_i \in D_i$. Suppose that $\sigma_i^*(c_i) = 0$ with positive probability. Then, by Lemma 3.3, the event $\sigma_{-i}^*(c_{-i}) = 0$ is null. Take some $c_{-i} \in D_{-i}$ with $\sigma_{-i}^*(c_{-i}) \neq 0$. Then, for any $c_i \in D_i$, by concavity, the difference quotient $\Pi_i(y_i, \sigma_{-i}^*(c_{-i}), c_i)/y_i$ is monotone increasing as $y_i \searrow 0$, with limit $\pi_i(\sigma^*, c_i, c_{-i})$. Since also $\Pi_i(y_i, \sigma_{-i}^*(c_{-i}), c_i)/y_i \geq -\overline{c}_i g_i'(E)$, the first claim follows from Levi's theorem. The second claim follows from Lebesgue's theorem, because $\overline{\pi}_i(\sigma^*, \cdot) \leq 0$ from the Kuhn–Tucker conditions, and because $\overline{\pi}_i(\sigma^*, \cdot) \geq -\overline{c}_i g_i'(E)$, as above. □

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